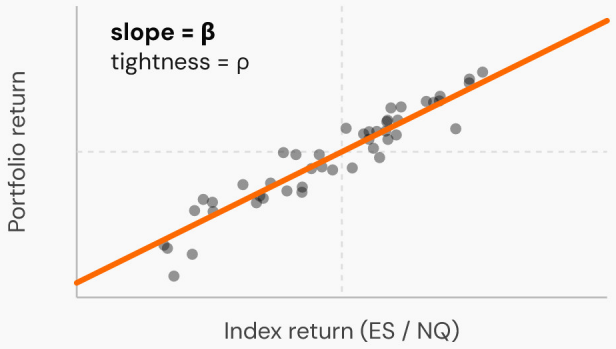
Correlation Considerations for Index & Single Stock Futures Hedges

A futures hedge only removes the part of your portfolio's risk that moves with the contract. **Beta (β)** sets how many contracts to short; **correlation (ρ)** sets how much risk that short actually removes.

Beta is the slope; correlation is the fit



Illustrative daily returns, not market data. Points far from the line are risk the index short cannot remove.

Hedge effectiveness = ρ^2



ρ	0.80	0.90	0.95
Removed	64%	81%	90%
Vol left	60%	44%	31%

The formulas

$$\beta = \mathrm{Cov}(r_P, r_I) \div \mathrm{Var}(r_I) = \rho \cdot \sigma_P \div \sigma_I$$

Portfolio beta to the index (ES or NQ)

$$\text{Contracts} = -\beta \cdot V \div (\text{multiplier} \cdot F)$$

One-index hedge; V = portfolio value, F = futures price

$$q^* = -(A^T \Sigma A)^{-1} A^T \Sigma x$$

Best-fit mix of several contracts (ES, NQ, single stocks)

$$R^2 = 1 - \mathrm{Var}(x + A \cdot q) \div \mathrm{Var}(x)$$

Hedge effectiveness after rounding to whole contracts

Key: x = dollar holdings · Σ = return covariance · A = each contract's exposure (notional × index weight; a single stock future loads only its own stock) · Multipliers: ES $50, MES $5, NQ $20, MNQ $2, single stock future 100 shares, micro 10 shares

What weakens the correlation

- → **ES and NQ move together.** Solving for both at once is unstable; regularize or fix the ES/NQ split by your tech weight.
- → **Correlations shift in sell-offs.** Test the hedge on a stress window, not only calm data.
- → **Estimation noise.** Use 60–250 trading days with a shrinkage estimator, and re-estimate on a set cadence.
- → **Stocks outside the index** are hedged only through their correlation, so check the risk left over.
- → **Basis, dividends, rolls and rounding** add tracking error; finish sizing with micros (1 ES = 10 MES).

How Claude CoWork confirms beta

1. **Connect.** Pull holdings and trade history from broker exports (such as NinjaTrader CSVs) and files in your connected drives.
2. **Fetch.** Gather price history from connected market-data sources and the web, plus CME contract specs.
3. **Calculate.** Recompute rolling β, ρ and R² for each contract and flag drift past your threshold.
4. **Deliver.** Write the contract mix into your trade-log workbook and a branded summary, on the schedule you set.

Michael Bardy
Michael Bardy LTD

Educational illustration, not investment advice. Futures and security futures involve substantial risk of loss and are not suitable for everyone. Prepared